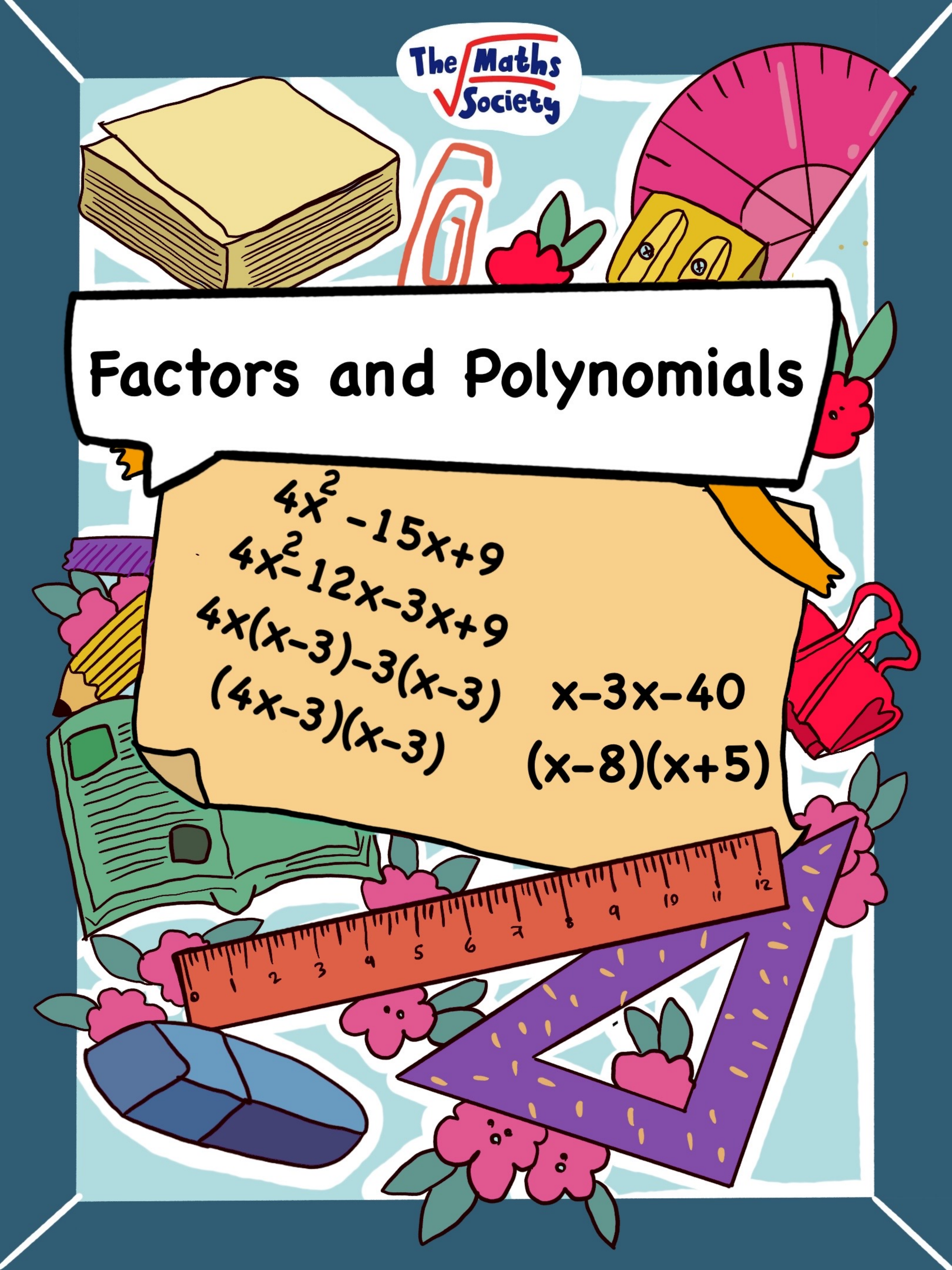
Factors and Polynomials



Chapter 3 - Factors and Polynomials

1. The three roots of $p(x) = 0$, where $p(x) = 5x^3 + ax^2 + bx - 2$ are $x = \frac{1}{5}$, $x = n$ and $x = n + 1$, where a and b are positive integers and n is a negative integer. Find $p(x)$, simplifying your coefficients.

[5]

$(5x-1)(x-n)(x-n-1)$

$-n^2-n = -2$

$n^2+n-2=0$

$(n+2)(n-1)=0$

$n=-2$ (or) $n=1$ (reject)

$(5x-1)(x+2)(x+2-1)$

$=(5x-1)(x+2)(x+1)$

$=(5x^2+10x-x-2)(x+1)$

$=(5x^2+9x-2)(x+1)$

$=5x^3+5x^2+9x^2+9x-2x-2$

$=5x^3+14x^2+7x-2$

$=5x^3+ax^2+bx-2$

$a=14$, $b=7$

2. The polynomial $p(x) = 6x^3 + ax^2 + 6x + b$, where a and b are integers, is divisible by $2x - 1$. When $p(x)$ is divided by $x - 2$, the remainder is 120.

(a) Find the values of a and b. [4]

$p(x) = 6x^3 + ax^2 + 6x + b$

$p\left(\frac{1}{2}\right) = 0$

$6\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + b = 0$

$\frac{3}{4} + \frac{1}{4}a + 3 + b = 0$

$\left(\frac{1}{4}a + b = -\frac{15}{4}\right) \times 4$

$a + 4b = -15$

$p(2) = 120$

$6(2)^3 + a(2)^2 + 6(2) + b = 120$

$48 + 4a + 12 + b = 120$

$4a + b = 60$

$a + 4b = -15$

$a = -15 - 4b$

$4(-15 - 4b) + b = 60$

$-60 - 16b + b = 60$

$-16b + b = 60 + 60$

$-15b = 120$

$b = -8$

$a = -15 - 4(-8)$

$= -15 + 32$

$= 17$

(b) Hence write down the remainder when $p(x)$ is divided by x. [1]

$$
\begin{array}{r|l}
 & 3x^2 + 10x + 8 \\
2x - 1 & 6x^3 + 17x^2 + 6x - 8 \\
 & 6x^3 - 3x^2 \\
\hline
 & 20x^2 + 6x \\
 & 20x^2 - 10x \\
\hline
 & 16x - 8 \\
 & 16x - 8 \\
\hline
 & 0
\end{array}
$$

$p(x) = (2x - 1)(3x^2 + 10x + 8)$

3. The polynomial $p(x) = mx^3 - 17x^2 + nx + 6$ has a factor $x - 3$. It has a remainder of -12 when divided by $x + 1$. Find the remainder when $p(x)$ is divided by $x - 2$.

[6]

$p(3) = 0$

$m(3)^3 - 17(3)^2 + n(3) + 6 = 0$

$27m - 153 + 3n + 6 = 0$

$27m + 3n = 147$

$p(-1) = -12$

$m(-1)^3 - 17(-1)^2 + n(-1) + 6 = -12$

$-m - 17 - n + 6 = -12$

$-m - n = -1$

$m + n = 1$

$m = 1 - n$

$27(1 - n) + 3n = 147$

$27 - 27n + 3n = 147$

$-24n = 120$

$n = -5$

$m = 1 - (-5)$

$= 6$

$p(x) = 6x^3 - 17x^2 - 5x + 6$

$p(2) = 6(2)^3 - 17(2)^2 - 5(2) + 6$

$= 48 - 68 - 10 + 6$

$= -24$

4. The polynomial $p(x)$ is such that $p(x) = 6x^3 + ax^2 - 52x + b$, where a and b are integers. It is given that $p(x)$ is divisible by $2x - 3$ and that $p'(1) = 4$.(Chapter 12 differentiation)

(a) Find the values of a and b. [5]

$$p\left(\frac{3}{2}\right) = 6\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 - 52\left(\frac{3}{2}\right) + b$$

$$= \frac{81}{4} + \frac{9}{4}a - 78 + b$$

$$\left(\frac{231}{4} = \frac{9}{4}a + b\right) \times 4$$

$$231 = 9a + 4b$$

$$p'(x) = 18x^2 + 2ax - 52$$

$$p'(1) = 4$$

$$18(1)^2 + 2a(1) - 52 = 4$$

$$18 + 2a = 56 \qquad 2a = 38$$

$$a = 19$$

$$231 - 9(19) = 4b$$

$$b = 15$$

DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

(b) Using your values of a and b, factories $p(x)$ fully. [3]

$$p(x) = 6x^3 + 19x^2 - 52x + 15$$

$$\begin{array}{r|l}
 & 3x^2 + 14x - 5 \\
\hline
2x - 3 & 6x^3 + 19x^2 - 52x + 15 \\
 & 6x^3 - 9x^2 \\
\hline
 & 28x^2 - 52x \\
 & 28x^2 - 42x \\
\hline
 & -10x + 15 \\
 & -10x + 15 \\
\hline
 & 0
\end{array}$$

$$\begin{array}{ccc} 3 & - & 1 \\ 1 & + & 5 \end{array}$$

$$p(x) = (2x-3)(3x^2+14x-5)$$

$$= (2x-3)(3x-1)(x+5)$$

5. The polynomial $p(x)$ is such that $p(x) = ax^3 + 13x^2 + bx + c$, where a, b and c are integers. It is given that $p'(0) =- 9$.(Chapter 12 differentiation)

(a) Show that $b =- 9$. [1]

$p'(x) = 3ax^2 + 26x + b$

$p'(0) = -9$

$3a(-9)^2 + 26(-9) + b$

$243 - 234 + b$

$b = -9$

It is also given that $3x + 2$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x + 1$ the remainder is 6.

(b) Find the values of a and c. [4]

$p(-\frac{2}{3}) = 0$

$a(-\frac{2}{3})^3 + 13(-\frac{2}{3})^2 + (-9)(-\frac{2}{3}) + c = 0$

$-\frac{8}{27}a + \frac{52}{9} + 6 + c = 0$

$(-\frac{8}{27}a + c = -\frac{106}{9}) \times 27$

$-8a + 27c = -318$

$p(-1) = 6$

$a(-1)^3 + 13(-1)^2 + (-9)(-1) + c = 6$

$-a + 13 + 9 + c = 6$

$-a + c = -16$ $\quad c = -16 + a$

$-8a + 27(-16 + a) = -318$

$-8a - 432 + 27a = -318$

$19a = 114$

$a = 6$

$c = -16 + 6$

$= -10$

(c) Find the quadratic $q(x)$ such that $p(x) = (3x + 2) \times q(x)$. [1]

$$
\begin{array}{r|l}
 & 2x^2 + 3x - 5 \\
3x + 2 & 6x^3 + 13x^2 - 9x - 10 \\
 & 6x^3 + 4x^2 \\
\hline
 & 9x^2 - 9x \\
 & 9x^2 + 6x \\
\hline
 & -15x - 10 \\
 & -15x - 10 \\
\hline
 & 0
\end{array}
$$

$q(x) = 2x^2 + 3x - 5$

$p(x) = (3x + 2)(2x^2 + 3x - 5)$

(d) Hence find $p(x)$ as a product of linear factors with integer coefficients. [1]

$p(x) = (3x + 2)(2x^2 + 3x - 5)$

$= (3x + 2)(2x + 5)(x - 1)$

2 5

1 1